

RECENT RESULTS FOR YANG-MILLS THEORY RESTRICTED TO THE GRIBOV REGION*

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We summarize recent results for the Gribov-Zwanziger Lagrangian which includes the effect of restricting the path integral to the first Gribov region. These include the two loop $\overline{\text{MS}}$ and one loop MOM gap equations for the Gribov mass.

Keywords: Gribov problem, renormalization.

1. Introduction

The generalization of quantum electrodynamics to include non-abelian gauge fields produces the asymptotically free gauge theory called quantum chromodynamics (QCD) which describes the strong interactions. The natural forum to construct the properly gauge fixed (renormalizable) Lagrangian with which to perform calculations, is provided by the path integral machinery. For instance in the Landau gauge, which we concentrate on here, the Faddeev-Popov ghosts naturally emerge as a consequence of the non-gauge invariance of the path integral measure. Whilst the resulting Lagrangian more than adequately describes the ultraviolet structure of asymptotically free quarks and gluons the infrared behaviour has not been fully established. For instance, it is evident that as a result of confinement gluons and quarks cannot have propagators of a fundamental type. Over the last few years there has been intense activity into measuring gluon and ghost form factors using lattice methods and the Dyson Schwinger formalism. Denoting these respectively by $D_A(p^2)$ and $D_c(p^2)$ a general picture emerges in that there is gluon suppression with $D_A(0) = 0$ and ghost enhancement where

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$D_c(p^2) \sim 1/(p^2)^\lambda$ as $p^2 \rightarrow 0$ with $\lambda > 0$. Such behaviour is not inconsistent with general considerations from confinement criteria^{1, 2, 3, 4, 5, 6, 7, 8, 9}. Ideally given that these properties are now accepted, it is important that they can be explained from general field theory considerations. This was the approach of Zwanziger^{4, 5, 7, 8} in treating the Gribov problem from the path integral point of view. Therefore we will briefly review the construction of the Gribov-Zwanziger Lagrangian before giving a summary of recent results of using it in the Landau gauge.

2. Gribov-Zwanziger Lagrangian

Gribov pointed out¹ that in non-abelian gauge theories it is not possible to uniquely fix the gauge globally due to the existence of copies of the gauge field. To handle this the path integral was restricted to the first Gribov region, Ω , where $\partial\Omega$ is defined by the place where the Faddeev-Popov operator $\mathcal{M} = -\partial^\mu D_\mu$ first vanishes. Within Ω \mathcal{M} is always positive and in the Landau gauge it is hermitian. Moreover Ω is convex and bounded³ and all gauge copies transit³ Ω . Any copy in the subsequent regions defined by the other zeroes of \mathcal{M} can be mapped into Ω . Whilst the path integral is constrained to Ω , within Ω there is a region, Λ , known as the fundamental modular region where there are no gauge copies and the gauge is properly fixed. Although Λ is difficult to define, for practical purposes expectation values over Λ or Ω give the same values¹⁰. Consequently the gluon form factor is modified to $D_A(p^2) = (p^2)^2/[(p^2)^2 + C_A\gamma^4]$ where γ is the Gribov mass, whence suppression emerges¹. The parameter γ is not independent and satisfies a gap equation. The theory can only be interpreted as a gauge theory when γ takes the value defined in the gap equation. Thence computing the one loop ghost propagator, it is enhanced precisely when the gap equation is satisfied¹.

Gribov's revolutionary analysis was based on a semi-classical approach and then Zwanziger^{4, 5} extended it to a path integral construction by modifying the measure to restrict the integration region to Ω via the defining criterion known as the horizon condition,

$$\int A_\mu^a(x) \frac{1}{\partial^\nu D_\nu} A^{a\mu}(x) = \frac{dN_A}{C_A g^2} \quad (1)$$

where d is the dimension of spacetime and N_A is the adjoint representation dimension⁵. For the Landau gauge the convexity and ellipsoidal properties of Ω allow one to modify the usual Yang-Mills Lagrangian to include the

horizon condition, (1), producing the non-local Yang-Mills Lagrangian^{4, 5}

$$L^\gamma = -\frac{1}{4}G_{\mu\nu}^a G^{a\mu\nu} + \frac{C_A\gamma^4}{2}A_\mu^a \frac{1}{\partial^\nu D_\nu} A^{a\mu} - \frac{dN_A\gamma^4}{2g^2}. \quad (2)$$

Again (2) only has meaning when γ satisfies (1) which is equivalent to the Gribov gap equation. Finally the non-locality can be handled by using localizing fields to produce the Gribov-Zwanziger Lagrangian⁵

$$\begin{aligned} L^Z = & L^{QCD} + \bar{\phi}^{ab\mu} \partial^\nu (D_\nu \phi_\mu)^{ab} - \bar{\omega}^{ab\mu} \partial^\nu (D_\nu \omega_\mu)^{ab} \\ & - g f^{abc} \partial^\nu \bar{\omega}_\mu^{ae} (D_\nu c)^b \phi_\mu^{ec} \\ & + \frac{\gamma^2}{\sqrt{2}} (f^{abc} A^{a\mu} \phi_\mu^{bc} + f^{abc} A^{a\mu} \bar{\phi}_\mu^{bc}) - \frac{dN_A\gamma^4}{2g^2} \end{aligned} \quad (3)$$

where ϕ_μ^{ab} and ω_μ^{ab} are localizing ghost fields with the latter anti-commuting. This Lagrangian is renormalizable^{7, 11, 12} and reproduces Gribov's one loop gap equation and ghost enhancement⁸. For (3) the horizon condition equates to

$$f^{abc} \langle A^{a\mu}(x) \phi_\mu^{bc}(x) \rangle = \frac{dN_A\gamma^2}{\sqrt{2}g^2}. \quad (4)$$

3. Calculations

As the Zwanziger construction has produced a renormalizable Lagrangian with extra fields incorporating infrared features without upsetting ultra-violet properties, such as asymptotic freedom, it is possible to extend the earlier one loop analysis^{1, 8}. For instance in $\overline{\text{MS}}$ the two loop gap equation results from (4) after computing 17 vacuum bubble graphs, giving¹³,

$$\begin{aligned} 1 = & C_A \left[\frac{5}{8} - \frac{3}{8} \ln \left(\frac{C_A\gamma^4}{\mu^4} \right) \right] a \\ & + \left[C_A^2 \left(\frac{2017}{768} - \frac{11097}{2048} s_2 + \frac{95}{256} \zeta(2) - \frac{65}{48} \ln \left(\frac{C_A\gamma^4}{\mu^4} \right) \right. \right. \\ & \quad \left. \left. + \frac{35}{128} \left(\ln \left(\frac{C_A\gamma^4}{\mu^4} \right) \right)^2 + \frac{1137}{2560} \sqrt{5} \zeta(2) - \frac{205\pi^2}{512} \right) \right. \\ & \quad \left. + C_A T_F N_f \left(-\frac{25}{24} - \zeta(2) + \frac{7}{12} \ln \left(\frac{C_A\gamma^4}{\mu^4} \right) \right. \right. \\ & \quad \left. \left. - \frac{1}{8} \left(\ln \left(\frac{C_A\gamma^4}{\mu^4} \right) \right)^2 + \frac{\pi^2}{8} \right) \right] a^2 + O(a^3) \end{aligned} \quad (5)$$

where $s_2 = (2\sqrt{3}/9)\text{Cl}_2(2\pi/3)$ with $\text{Cl}_2(x)$ the Clausen function, $\zeta(n)$ is the Riemann zeta function and $a = \alpha_S/(4\pi)$. To appreciate the non-perturbative nature of γ one can formally solve for it with the ansatz

$$\frac{C_A \gamma^4}{\mu^4} = c_0 [1 + c_1 C_A \alpha_S] \exp \left[-\frac{b_0}{C_A \alpha_S} \right] \quad (6)$$

giving

$$b_0 = \frac{32\pi \left[3C_A - \sqrt{79C_A^2 - 32C_A T_F N_f} \right]}{[35C_A - 16T_F N_f]} \quad (7)$$

$$c_0 = \exp \left[\frac{1}{[105C_A - 48T_F N_f]} \left[260C_A - 112T_F N_f - \frac{[255C_A - 96T_F N_f]C_A}{\sqrt{79C_A^2 - 32C_A T_F N_f}} \right] \right] \quad (8)$$

and

$$\begin{aligned} c_1 = & \left[8940981420\sqrt{5}C_A^4\zeta(2) - 11330632512\sqrt{5}C_A^3N_fT_F\zeta(2) \right. \\ & + 4778237952\sqrt{5}C_A^2N_f^2T_F^2\zeta(2) - 670629888\sqrt{5}C_A N_f^3T_F^3\zeta(2) \\ & - 8060251500\pi^2C_A^4 - 109078793775s_2C_A^4 \\ & + 7470477000C_A^4\zeta(2) + 19529637400C_A^4 \\ & + 12730881600\pi^2C_A^3N_fT_F + 138232221840s_2C_A^3N_fT_F \\ & - 29598076800C_A^3N_fT_F\zeta(2) - 32025280640C_A^3N_fT_F \\ & - 7496478720\pi^2C_A^2N_f^2T_F^2 - 58293872640s_2C_A^2N_f^2T_F^2 \\ & + 29503733760C_A^2N_f^2T_F^2\zeta(2) + 19655024640C_A^2N_f^2T_F^2 \\ & + 1949368320\pi^2C_A N_f^3T_F^3 + 8181596160s_2C_A N_f^3T_F^3 \\ & - 11318722560C_A N_f^3T_F^3\zeta(2) - 5351014400C_A N_f^3T_F^3 \\ & \left. - 188743680\pi^2N_f^4T_F^4 + 1509949440N_f^4T_F^4\zeta(2) + 545259520N_f^4T_F^4 \right] \\ & \times \frac{1}{46080\pi[79C_A - 32T_F N_f]^{5/2}[35C_A - 16T_F N_f]\sqrt{C_A}}. \quad (9) \end{aligned}$$

So in principle one could now compute with a gluon propagator which includes renormalon type singularities. Further, with (5) there is two loop ghost enhancement with the Kugo-Ojima confinement criterion⁹ precisely fulfilled at this order consistent with Zwanziger's all orders proof⁷. Also at one loop it has been shown¹⁴ that $D_A(0) = 0$. The final quantity of interest is the renormalization group invariant effective coupling constant $\alpha_S^{\text{eff}}(p^2) = \alpha_S(\mu)D_A(p^2)(D_c(p^2))^2$ which is believed to freeze at

zero momentum. From the $\overline{\text{MS}}$ one loop form factors it was shown¹⁴ that $\alpha_S^{\text{eff}}(0) = \frac{50}{3\pi C_A}$.

Whilst the previous expressions have all been in the $\overline{\text{MS}}$ scheme it is worth considering other renormalization schemes such as MOM. Given that one loop calculations¹⁴ produced exact form factors the derivation of the one loop MOM gap equation is straightforward, giving

$$1 = \left[\frac{5}{8} + \frac{3}{8} \ln \left(\frac{C_A \gamma^4}{[C_A \gamma^4 + \mu^4]} \right) - \frac{C_A \gamma^4}{8\mu^4} \ln \left(\frac{C_A \gamma^4}{[C_A \gamma^4 + \mu^4]} \right) - \frac{3\pi\sqrt{C_A}\gamma^2}{8\mu^2} \right. \\ \left. + \left[\frac{3\sqrt{C_A}\gamma^2}{4\mu^2} - \frac{\mu^2}{4\sqrt{C_A}\gamma^2} \right] \tan^{-1} \left[\frac{\sqrt{C_A}\gamma^2}{\mu^2} \right] \right] C_A a + O(a^2). \quad (10)$$

For later we formally define this as $1 = \text{gap}(\gamma, \mu, \text{MOM}) C_A a + O(a^2)$. Central to deriving this was the preservation of the Slavnov-Taylor identities in MOM. For instance defining Z_A and Z_c from the respective gluon and ghost 2-point functions in MOM, then the coupling constant and γ renormalization constants are already fixed and these must be used in computing the horizon function. Given (10) we have reproduced the one loop ghost enhancement in MOM and the *same* freezing value for $\alpha_S^{\text{eff}}(0)$. Since the numerical structure is different from the $\overline{\text{MS}}$ calculation we record the analogous¹⁴ computation is

$$\alpha_S^{\text{eff}}(0) = \lim_{p^2 \rightarrow 0} \left[\frac{\alpha_S(\mu) \left[1 - C_A \left(\text{gap}(\gamma, \mu, \text{MOM}) + \frac{5}{8} - \frac{265}{384} \right) a \right] (p^2)^2}{C_A \gamma^4 \left[1 - C_A \left(\text{gap}(\gamma, \mu, \text{MOM}) - \frac{\pi p^2}{8\sqrt{C_A}\gamma^2} \right) a \right]^2} \right] \quad (11)$$

whence $\alpha_S^{\text{eff}}(0) = \frac{50}{3\pi C_A}$.

4. Discussion

To conclude we note that we have reviewed the path integral construction of Zwanziger's localised renormalizable Lagrangian for the Landau gauge which incorporates the restriction of gauge configurations to the first Gribov region. A picture emerges of the infrared structure which is consistent with the gluon being confined. Crucial to the analysis was the geometry of the Gribov region. This can be appreciated from another point of view given recent work in trying to extend the path integral construction to other gauges^{15, 16, 17}. For linear covariant gauges other than Landau the Fadeev-Popov operator is not hermitian¹⁵ and convexity of the Gribov region is only valid when the covariant gauge fixing parameter is small¹⁵. Moreover, given that the Faddeev-Popov operator in this instance would

involve the transverse part of the gauge field then the non-local operator of (2) would itself contain a non-locality in the covariant derivative¹⁵. Another example is the construction of a Gribov-Zwanziger type Lagrangian for $SU(2)$ Yang-Mills fixed in the maximal abelian gauge^{16, 17}. Whilst a localised renormalizable Lagrangian analogous to (3) can be constructed the algebraic renormalization analysis demonstrates that there is an additional free parameter which has no analogue in the Landau gauge¹⁷. Given these recent considerations it would seem therefore that in the Gribov context the Landau gauge is peculiarly special.

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